The Performance Potential of Semisubmerged Ships

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As a contribution to the current debates about the potential of semisubmerged ships (S^3), this paper presents estimates of the lowest possible resistance/displacement ratio. That is, the resistance of the submerged form alone is calculated, without reference to the additional components due to necessary struts, rudders, and other appendages. It is suggested that the Priest hull configuration is better than the more recent SWATH, because of its reduced wetted area, and that, like hydrofoil and SES, S^3 should probably rise up in the water for high-speed "flight." This then removes the serious constraint that the struts must provide adequate hydrostatic stability, since this stability can now be provided by the upper hull, which is in contact with the water at low speeds. At higher speeds, roll stability can be provided by the submerged foils which in any case, seem to be required for adequate control.

Introduction—A Brief History of Semisubmerged Ships

SINCE there does not seem to be any totally satisfactory name for semisubmerged ships, the abbreviation S^3 is used because of its brevity. The real attempt in this paper has been to categorize all ships which have low self-making wave drag, *and* reduced response to the seaway by virtue of the fact that most of their buoyant volume is submerged, and their waterplane area very small. A "small waterplane area" ship generally will have both these virtues, but the description is imprecise. Also, there is at least one other way of achieving the same result, which does not have either a small waterplane area or a majority of hull submerged, but does have a low waterplane moment of area in pitch.

The first patent for S^3 was awarded to Reuben N. Perley (1922) for a submerged monohull with surface-piercing struts forward and aft. A similar ship with a single strut (the "Shark form") was patented by Rudolf Engleman in 1937. A modern SWATH configuration was patented (in England) by Frederick G. Creed in 1944. Thus, there is nothing particularly new about the basic idea or concept. The problem has been in assessing its virtues and vices.

It would seem that the earliest studies in this country were carried out by a group in BuShips (Code 420) under Owen Oakley, in the early fifties. Boericke[1] summarizes much of the work done by this creative team, which included Robert W. Priest, David Winter, James L. Mills, and others. Unfortunately, they were apparently unable to arouse the necessary interest for the suport of the broad-based R&D effort which would have been required to select and develop the best among their various proposed configurations. Figure 1 depicts some of the configurations they studied.

From a different vantage point, Lewis' studies[2] of seakeeping in the mid-fifties led to the first semisubmerged ship (S^3) described as such, and a form which he patented in 1959.[3] The lines are given in Fig. 2. This apparently was the first time that a form had been consciously designed for supercritical operation into head seas; although supercritical operation was known to be possible with conventional forms under certain rather special conditions. In so far as one can tie it down to one individual, Lewis was clearly the inventor of the supercritical ship.

The work of this intellectually fruitful decade was summarized in a number of important papers which appeared in the period from 1959-1962. Studies done by Mandel[4] and Lewis and Breslin[5] are the best and the most complete. But Mandel's statement that "none of the new ship types... is likely to supplant more traditional ships and aircraft..." may have discouraged some further work, because it was so easily taken out of context.

At this point, one might say that the most severe seakeeping problem—high speed in head seas—had been solved by Lewis' invention of the supercritical ship (SCS) and that this concept could be applied to some of the configurations already proposed. But the monohull SCS was very tender in roll, had insufficient deck space for many purposes, and experienced rather extreme motions at resonance.

In the period from 1964 to 1966, Payne, Inc., built two small, manned, supercritical catamarans (FICAT I and II) which avoided the first two of these three problems. The first of these craft is shown in Fig. 3. A reduction in wave drag was partially achieved by the reduction in individual hull beam

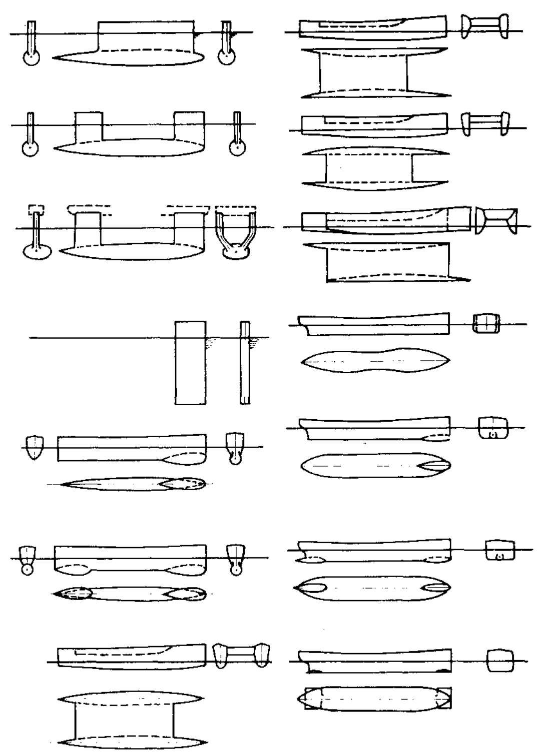

Fig. 1 Configurations studied by the Code 420 (preliminary design) team in the early fifties (from Boericke[1]).

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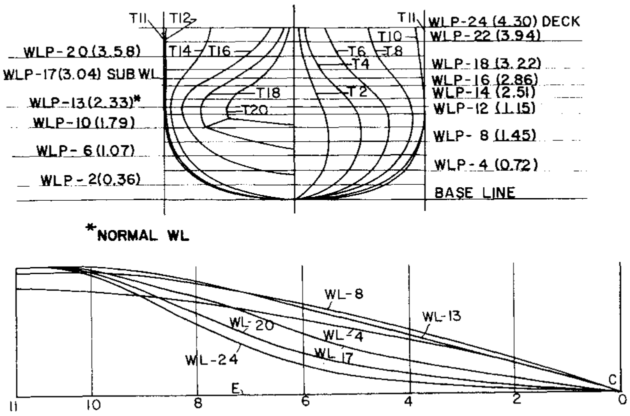

Fig. 2 Lines of Lewis' semisubmerged ship.[3]

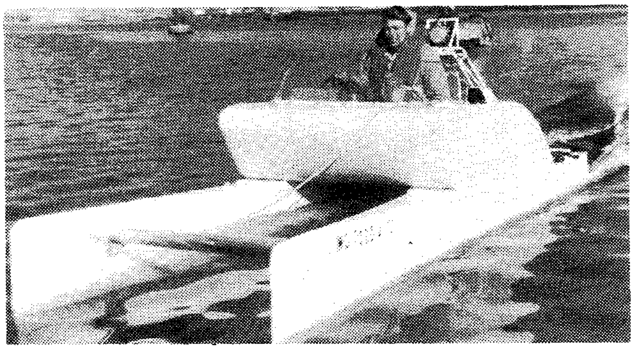

Fig. 3 FICAT I model with air propulsion (1965).

which is permitted by the catamaran configuration, and partly by cancellation interference of the wave trains from each hull. In other words, interference was substituted for submergence of the main buoyancy volume. This avoided some of the practical problems associated with S^3 struts, excessive draft, etc., and resulted in a more conventional ship.

The two FICAT's were operated "at sea" under widely varying conditions in the Chesapeake area for some hundreds of hours from 1965 to 1966. Model tests (Fig. 4) were also conducted, and it seemed clear that at least a 50% wave-drag cancellation was being achieved for all Froude numbers above about 0.5, with some evidence of total cancellation at $F = 0.7$.[6] The FICAT's were the first SCS configurations to be analyzed theoretically[7-9] so far as resistance was concerned. Band[9] found generally good agreement between Michell's wave-drag integral and experiment, including wave drag calculated from photographs of the local surface elevation. FICAT II, shown in Fig. 5, was driven by a small aircraft engine and propeller to permit resistance measurements via a load cell. The version shown in the figure, circa late 1966, was the last one tested.

Because Payne, Inc., was unable to find support for its research, it was discontinued in early 1969 so that attention could be concentrated on supercritical planing hulls. A definitive assessment of FICAT vis-a-vis other configurations was never made, and so this must remain a question mark.

Coincident with Payne's departure from the field, Leopold[10,11] reintroduced the Creed configuration. Marbury[12] subsequently designed and built a two-man Trisec. This was the first true S^3 actually reduced to practice, and its seakeeping ability was impressive. The problems of dynamic instability and excessive motions near resonance were solved by using inclined active foils which acted as both rudders and pitch angle control, following the pioneering work of Lang, mentioned below.

Litton was also unsuccessful in obtaining funding to pursue this research, but the basic concept has since been an ongoing project in the Navy, principally at NSRDC and NAVSEC.

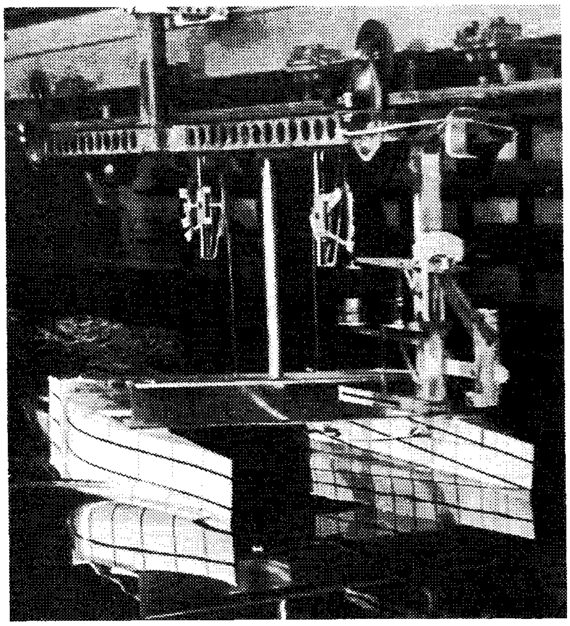

Fig. 4 A model FICAT experiencing bow "dig in" during tank testing. This is a form of pitch instability associated with suction loads on the bottoms of the floats. The floats have been reversed for this experiment at the Webb Institute (1966).

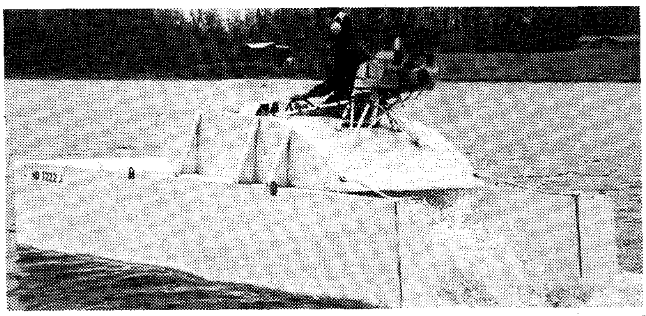

Fig. 5 FICAT II at speed on calm water. As the first manned supercritical ships, the FICAT's demonstrated the practicality of the concept under the scaled "real world" conditions in the Chesapeake Bay.

Shortly after Leopold, Lang[13,14] patented some very innovative improvements, and was later able to build the 190-ton "SSP Kaimalino," as a range support craft for NUC, which has been in use for that purpose since December 1975. The goals and boundary conditions for the SSP were as follows: 1) Near-level ride in large waves when underway; 2) low motion at rest; 3) manned access into the bow and stern regions of the submerged hulls; 4) plexiglass dome for underway viewing and research to be mounted at the bow of one submerged hull; 5) drag equal to, or less than, conventional monohulls at the 25-knot design speed; 6) stable, smooth operation at all speeds without automatic control; 7) a well in the center for stably raising and lowering up to 35-ton loads into the ocean in high-sea states; 8) a cover for the well so helicopters can land and take off in large waves, either at rest or underway; 9) adequate static stability, and sufficiently small waterplane area, so that up to 35-ton loads can be accommodated on top of the upper deck in large waves without excessive craft motion; and 10) dual-propulsion systems to improve reliability at sea, and to provide good maneuverability at rest and at low speeds.

This craft was reportedly successful in meeting its goals, and is therefore the first operational S^3. Lang was the originator of the active foil stabilization concept, and linked it to a simple autopilot to achieve minimum motions at all speeds.

Resistance Calculations

A key question in assessing the practicability of S^3 is the resistance/displacement ratio, and the fundamental component of this is the resistance of the underwater form. Without struts, rudders, stabilization foils, etc., this constitutes the irreducible "lower limit" which is the main subject of this paper. The basic theoretical tools for such an overview assessment of S^3 were developed by Havelock.[15-18]

For the present paper, we are interested in the wave resistance component only, the equations for which are, after considerable manipulation, as follows for the generalized ellipsoid[16] having semi-axes a, b, and c. If $a<b<c$

$$\frac{R_W}{\Delta}=\frac{24\pi\hat{b}\hat{c}}{(2-\alpha_0)^2(1-\hat{b}^2)^{3/2}}e^{-2/f^2}[\mathrm{Int}(1)+\mathrm{Int}(2)] \quad (1)$$

where

$$\hat{b}=b/a,\ \hat{c}=c/a,\ \hat{\lambda}=\lambda/a^2,\ \hat{h}=h/a$$

$$\alpha_0=\hat{b}\hat{c}\int_0^{\hat{\lambda}_1}\frac{\hat{\lambda}d\hat{\lambda}}{\sqrt{(1+\hat{\lambda})^3(\hat{b}^2+\hat{\lambda})(\hat{c}^2+\hat{\lambda})}}+\frac{2}{3}\hat{b}\hat{c}\hat{\lambda}_1^{-3/2} \quad (2)$$

$$\mathrm{Int}(1)=\frac{2}{\pi}\int_0^{1/\alpha_1}\frac{e^{-2t^2/f^2}}{q}\frac{[(\sin q)/q-\cos q]^2}{(1-\alpha_1^2t^2)^{3/2}}dt \quad (3)$$

where

$$q=\sqrt{\frac{(1-\hat{b}^2)(1+t^2)(1-\alpha_1^2t^2)}{f^4\hat{h}^2}} \quad (4)$$

$$\mathrm{Int}(2)=\frac{2}{\pi}\int_{1/\alpha_1}^{\infty}\frac{e^{-2t^2/f^2}}{p}\frac{\left[\cosh(p)-\dfrac{\sinh(p)}{p}\right]^2}{(\alpha_1^2t^2-1)^{3/2}}dt \quad (5)$$

If $a>c>b$

$$\frac{R_W}{\Delta}=\frac{24\pi\hat{b}\hat{c}e^{-2/f^2}}{(2-\alpha_0)^2(1-\hat{b}^2)^{3/2}}\mathrm{Int}(3) \quad (6)$$

where

$$\mathrm{Int}(3)=\int_0^{\infty}\frac{[J_{3/2}(r)]^2e^{-2t^2/f^2}}{(1+\alpha_2^2t^2)}dt$$

$$=\frac{2}{\pi}\int_0^{\infty}\frac{e^{-2t^2/f^2}}{r}\frac{\left(\dfrac{\sin r}{r}-\cos r\right)^2}{(1+\alpha_2^2t^2)^{3/2}}dt \quad (7)$$

where

$$\alpha_2^2=\frac{\hat{c}^2-\hat{b}^2}{1-\hat{b}^2}$$

and

$$r=\sqrt{\frac{(1-\hat{b}^2)(1+t^2)(1+\alpha^2t^2)}{f^4\hat{h}^2}} \quad (8)$$

Since the step from Havelock's equations to numerical results is not entirely trivial, we first checked our results with the only known previous solution, the "slender body" numerical evaluation by Wigley.[19] As Fig. 6 shows, the agreement is good, except at the "humps," which is what one would expect from the approximations made by Wigley. As recounted in the Appendix, we then compared the theory with tank test measurements of residual resistance; again with satisfactory agreement for our present purposes.

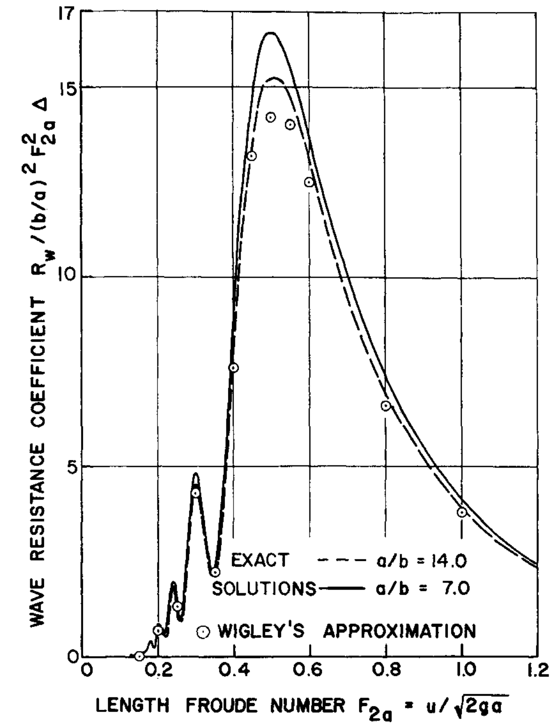

Fig. 6 Wigley's "slender body" numerical approximation in comparison with the exact solution of Havelock's equation for the wave drag of a prolate ellipsoid.

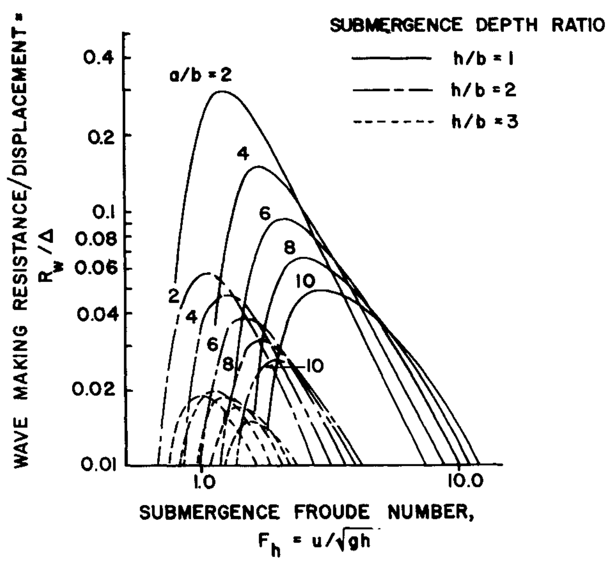

Fig. 7 The effect of slenderness and submergence on the wave drag of a prolate ellipsoid. h/b = submergence of centerline/radius of body

Some general trends were then studied. Figure 7 gives numerical results for a body of revolution and Fig. 8 shows the effect of varying the cross-sectional shape for a fixed submergence of the centerline. The corresponding total resistance ratio is given in Fig. 9 and the inverse, L/D, in Fig. 10. Changing the cross-sectional shape clearly has a very small effect compared with a change in depth, so this effect was ignored in subsequent calculations.

Before applying these results, we must ask whether the "conventional" twin-hull S^3 is really necessary. Given that the motion in a seaway can be very small, then the "lift/drag" (L/D) parameter (the inverse of resistance/displacement) which now seems the conventional efficiency parameter for advanced marine vehicles is by far the most important factor in any comparison with other forms of vehicle. That motions

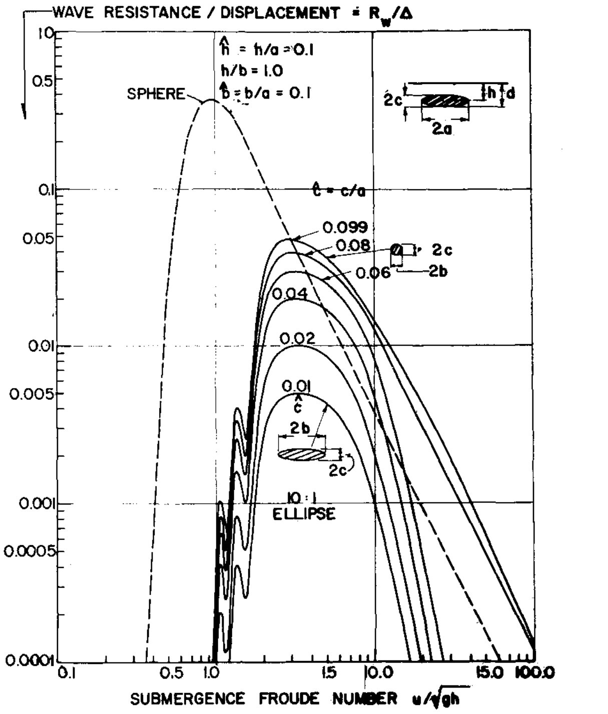

Fig. 8 The effect of varying the cross section of a submerged ellipsoid.

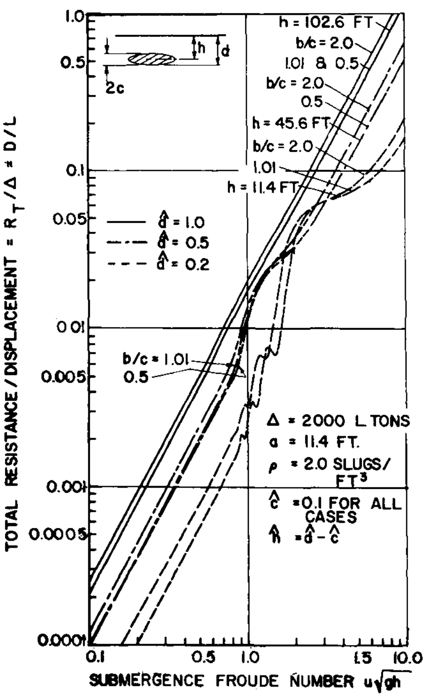

Fig. 9 Total resistance of immersed ellipsoids at constant draft, *d*.

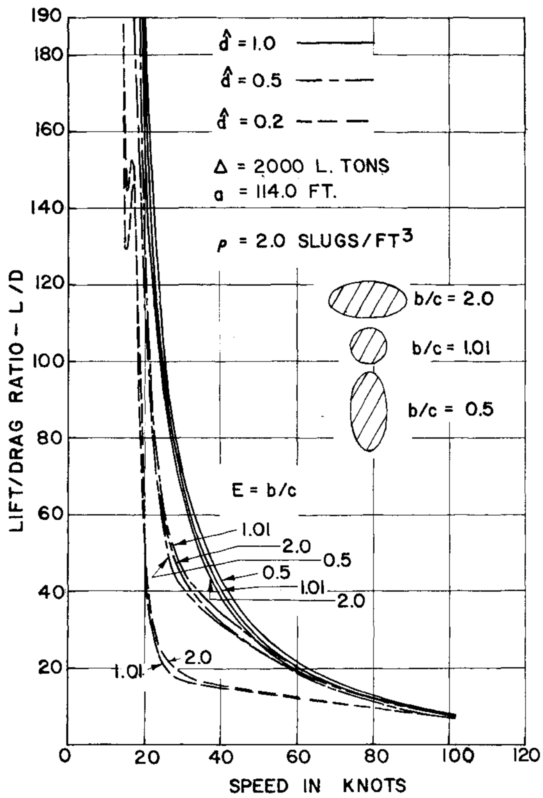

Fig. 10. Efficiency of submerged ellipsoids at constant draft.

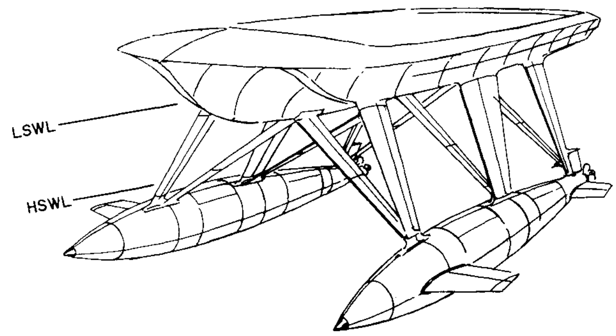

Fig. 11 A small waterplane twin hull (SWATH) configuration which becomes "foil-borne" for high-speed operation.

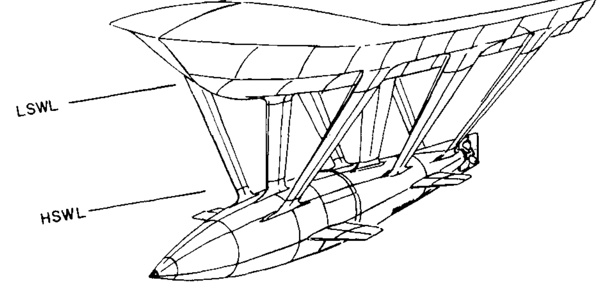

Fig. 12 A single submerged body configuration.

can be less than for conventional ships has been clearly shown by the work cited.

We suggest here, in fact, that motions could be essentially zero, if one is prepared to accept a form which has negligible hydrostatic pitch and roll stiffness when underway, relying on dynamic pressure forces on stabilizer foils for stability. The struts which support the above-water portion of the ship can then be sized by structural considerations only. Of course, this implies that the upper hull will descend to the water surface for low-speed operation, as indicated in Fig. 11. In many ways, operation of the Fig. 11 configuration would be similar to operation of a hydrofoil, and there are some interesting and potentially rewarding tradeoffs to be made between buoyant and dynamic lift. Note also that the ability to employ a multiplicity of struts means that the upper size limitations of a conventional hydrofoil are evaded.

The S^3 configuration in Fig. 11 has $\sqrt{2}$ times more wetted area than the minimum wetted area possible, so we show an alternative configuration in Fig. 12. This was first suggested

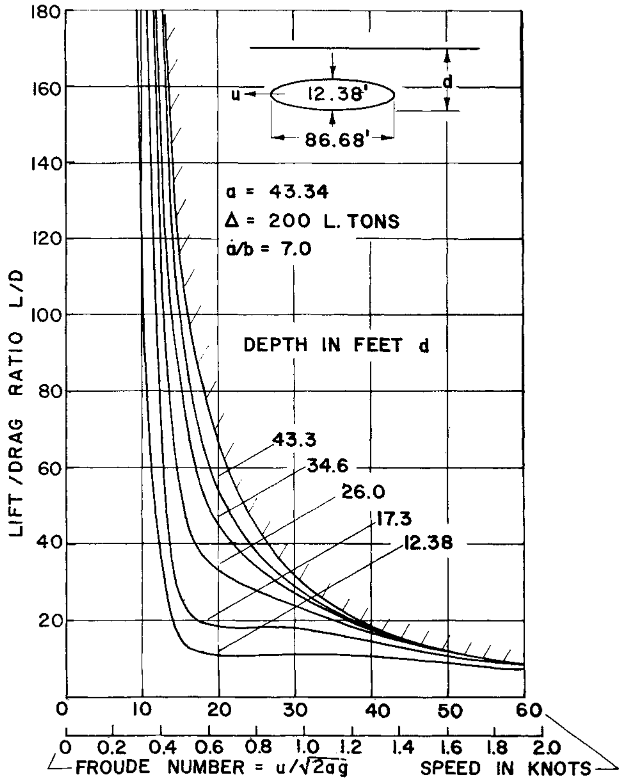

Fig. 13 Performance of a 200-ton submerged body.

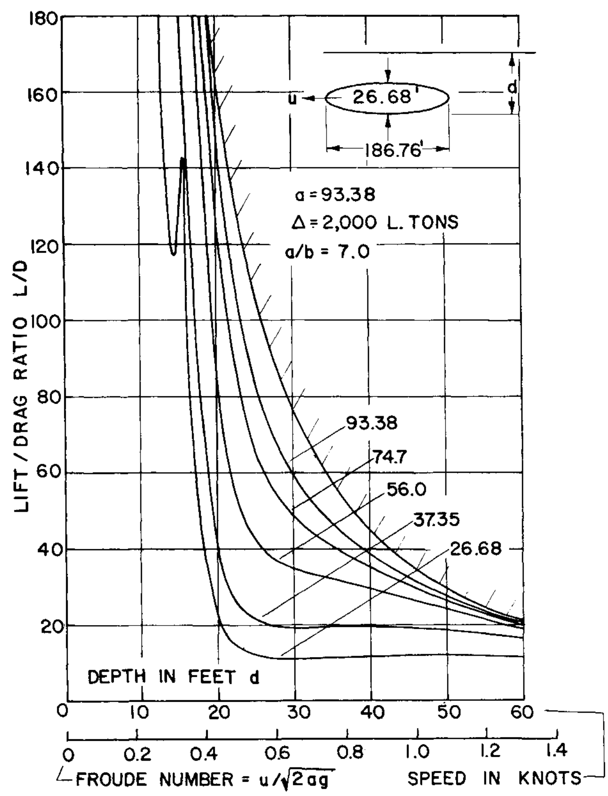

Fig. 14 Performance of a 2000-ton submerged body.

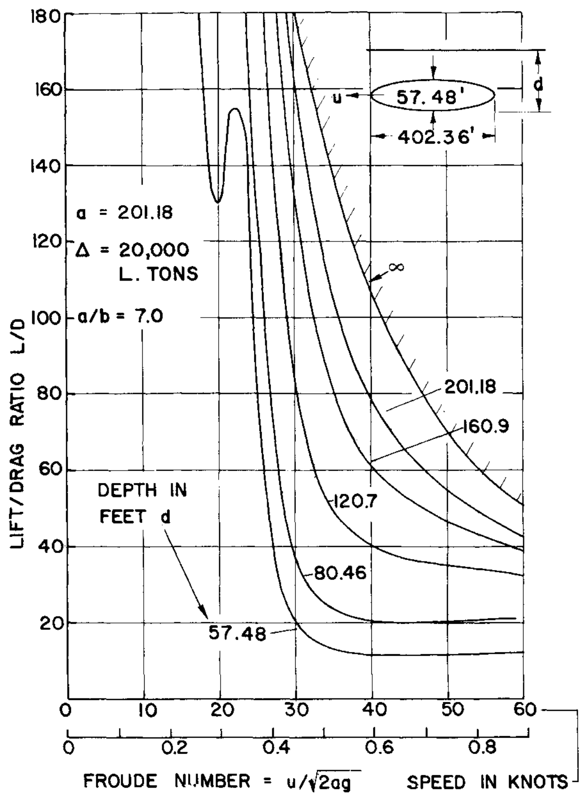

Fig. 15 Performance of a 20,000-ton submerged body.

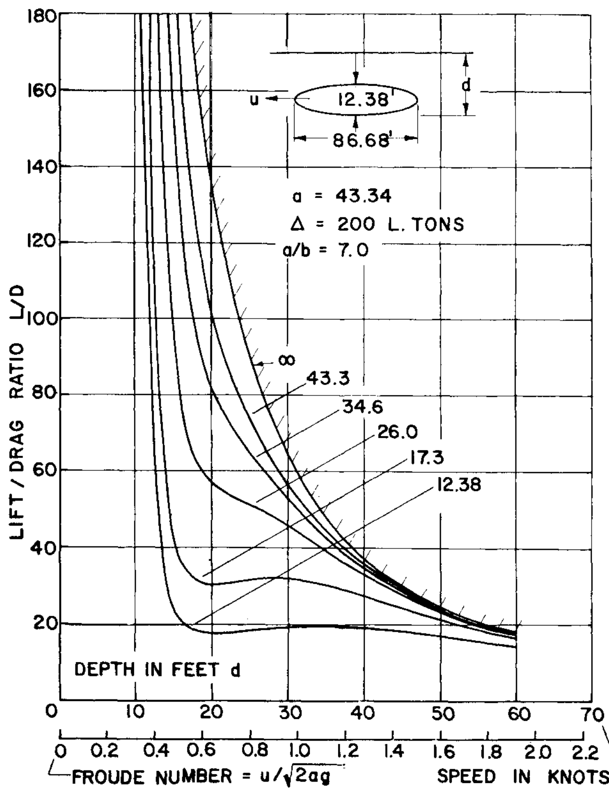

Fig. 16 Performance of a 200-ton submerged body if the skin-friction coefficient is halved.

by R.W. Priest in the fifties, (see Fig. 1, top left), although with struts large enough to provide hydrostatic stability.

The hydrodynamic efficiency (L/D) of such a single body is shown in Figs. 13-18, based on calculations described in the Appendix. If we select 60 knots as the speed of interest, we see that L/D increases markedly with size, but also with depth of immersion in the larger sizes. For $\Delta = 2000$ tons, $L/D \simeq 10$ with the top of the body at the surface; $\simeq 20$ with the top of the body 48 ft below the surface. For $\Delta = 20{,}000$ tons, there is not much difference when the draft is shallow, but a total draft of 161 ft gives $L/D = 39$. $L/D = 50$ is theoretically attainable, if the body is deep enough. These figures are very

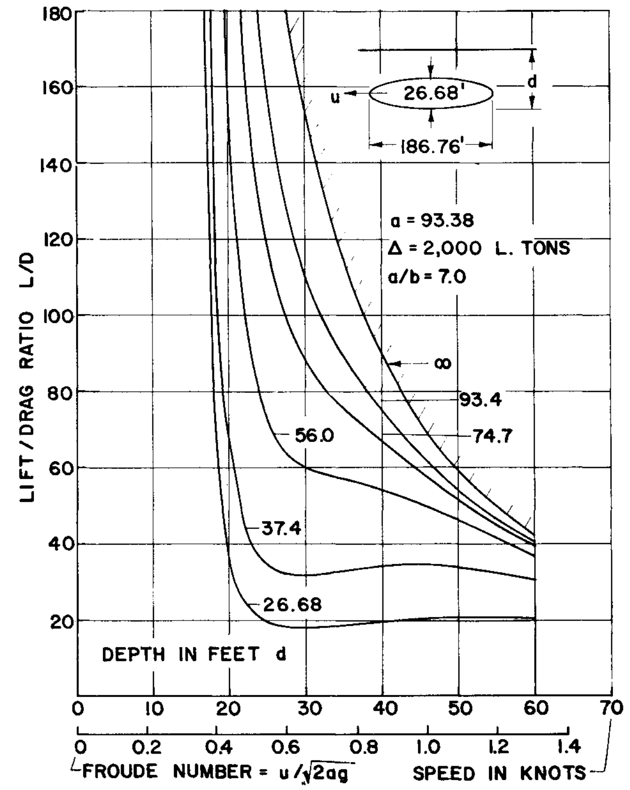

Fig. 17 Performance of a 2000-ton submerged body if the skin-friction coefficient is halved

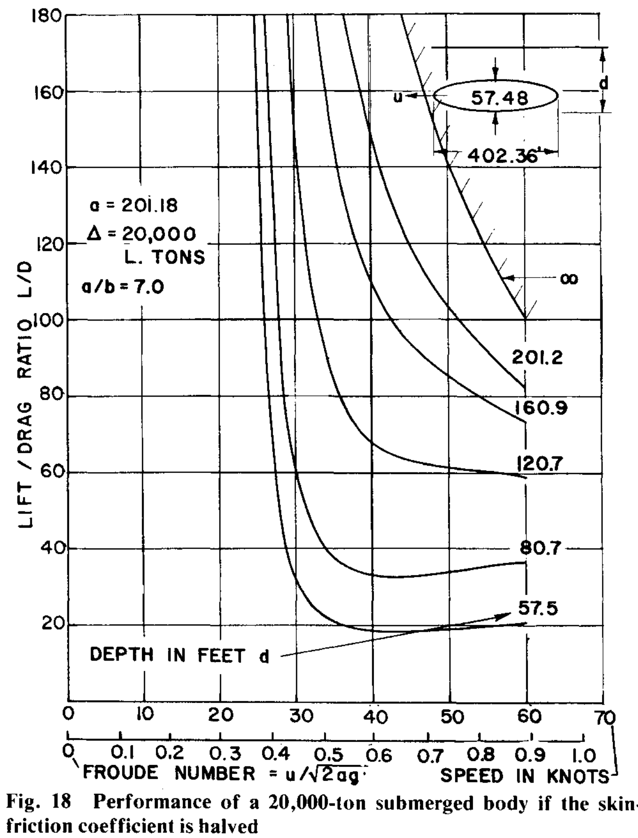

Fig. 18 Performance of a 20,000-ton submerged body if the skin-friction coefficient is halved

attractive by comparison with other kinds of advanced vehicle, where $L/D = 20$ is often a target and $L/D = 10$ is the best achieved to date.

So far, we have only considered conventional underwater bodies. Suppose it were possible to halve the skin-friction coefficient in some way; either by polymer injection (see Van Mater[20] for an example) or by appropriately shaping the body. The ultra-low-drag underwater bodies† are an example of such drag reduction by shaping, by virtue of extensive laminar flow in their boundary layers. Perhaps a more practical possibility for S^3 is to design for a "tired," low-friction turbulent boundary layer over most of the body. Stratford[22] has demonstrated such a flow experimentally, and while no one has yet investigated the technology exhaustively for external flows, to do so might prove fruitful. Other possibilities exist and have been discussed in the literature (e.g., moving belts, compliant skin) but our purpose here is not to say how it might be done, but to see what effect it would have if it could be achieved.

If it could be achieved in some way, then the L/D ratios for the three different displacements become as shown in Figs. 16-18. Even the 200-ton size is now competitive with other vehicles at 60 knots ($L/D \simeq 17$) while values as high as $L/D = 80$ are feasible in the 20,000-ton size. It should be emphasized that all of these values will be degraded by: strut drag; foil drag; and resistance of any other appendages. Since strut size dominates strut drag, and strut size depends principally on structural loads, we have not attempted to include estimates for these parasitic items in this paper.

Conclusions

It is suggested that in medium and large sizes, S^3 offers the possibility of high L/D ratios in the intermediate speed range around 60 knots, and large S^3 may be competitive at 100 knots. The poor showing to date seems to be due partly to designs which have the submerged hull too close to the surface, struts which are very thick and/or have large wetted area, and insufficient development work on reducing interference drag at the strut/hull intersection. But the poor showing is principally due, of course, to the very limited amount of research so far carried out.

It is also suggested that, since active stabilization is probably essential while underway, one might as well dispense with hydrostatic stability and thus avoid the rather severe resistance penalties which it imposes. We also suggest that the hulls should be much deeper in the water if high efficiency is to be achieved. Then , as a final improvement, the operational limitations imposed by this deep draft might be ameliorated by hinging the strut assembly in such a way that the above and underwater hulls can be brought together for such low-speed low-draft operation. Although there may be some size limitations to this last suggestion, the basic technology is not far removed from that used in the new variable-wing-sweep bomber and fighter aircraft.

Appendix: Approximate Calculation of Submerged Hull Resistance

In considering the feasibility of a new ship concept, an early question is "What is its transport efficiency?" We typically evaluate the calm water "lift/drag ratio" (the inverse of the more conventional R/Δ) in order to answer this, and it is clear that approximate figures for "L/D" are quite adequate for establishing feasibility so long as they are realistic. In the case of S^3, it is rather simple to compute resistance, and it is also clear that, due to small motions in a seaway, performance will not degrade much with increasing sea state; except possibly near resonance.

The significant resistance components are as follows:

R_W = hull wave-making drag
D_{BS} = hull friction drag, hull pressure drag, (considered together in this analysis)
D_{SW} = strut wave-making and spray drag
D_{SS} = strut skin-friction drag, strut pressure drag
D_i = strut/hull interference drag
D_A = wind resistance

†Payne[21] describes the antecedents of this technology.

The purpose of this appendix is to present equations for the first three resistance components.

Hull Wave-Making Drag

This is perhaps the most critical term, because it will dominate the optimization of strut length (a tradeoff which does not seem to have been addressed in the literature). On the other hand, it would be needlessly complicated to determine the wave drag of different hull shapes at this stage in the analysis, when we are looking for overall trends. It is sufficient to determine a "standard" variation of wave drag with slenderness ratio and immersion depth, recognizing that subsequent variations of hull-volume distribution may enable us to improve on this result.

Such a "standard" variation is provided by Havelock's analysis of a prolate spheroid[16] where he obtains

$$R_W = 128\pi^2 g\rho a^3 \epsilon^3 A^2 e^{-2/f^2} \int_0^\infty e^{-2t^2/f^2} [J_{3/2}(z)]^2 \mathrm{d}t \tag{A1}$$

where

ϵ $= \sqrt{1-(b/a)^2}$ the eccentricity
a, b = major and minor semi-axis lengths
f $= u/\sqrt{gh}$, the submergence Froude number
h = submergence of the spheroid centerline
$J_{3/2}(z)$ = the Bessel function of the first kind, of order 3/2

$$= \sqrt{\frac{2}{\pi z}}\left(\frac{\sin z}{z} - \cos z\right)$$

$$A = \left[\frac{4\epsilon}{1-\epsilon^2} - 2\log\frac{1+\epsilon}{1-\epsilon}\right]^{-1}$$

$$z = \frac{\epsilon}{f^2}\left(\frac{a}{h}\right)\sqrt{1+t^2}$$

The displacement of a prolate ellipsoid is

$$\Delta = \rho g \frac{4}{3}\pi a b^2 \tag{A2}$$

Thus, from Eqs. (A1) and (A2)

$$\frac{R_W}{\Delta} = \left\{96\pi\left(\frac{a}{b}\right)^2 \epsilon^3 A^2\right\} e^{-2/f^2} \frac{2}{\pi} \times \int_0^\infty \frac{e^{-2t^2/f^2}}{z}\left[\frac{\sin z}{z} - \cos z\right]^2 \mathrm{d}t \tag{A3}$$

Equation (A3) is easy to integrate numerically as it stands, so that there is little point in seeking further simplification.‡ In Fig. A1, we have compared it with some experimental measurements of net residual resistance from Appendix 7 of Ref. 23. Since the experimental data is for "streamline" bodies, the aft portions of which are quite unlike an ellipsoid, the agreement seems better than we might have expected up to the point of maximum wave-making resistance, including the magnitude of the peak value. The theoretical resistance then falls off much more rapidly than the experimental data, presumably because of the differences in bow and stern shapes of the test models, and/or viscous wake effects.

Hull Friction and Pressure Drag

Hoerner[24] gives a relationship for this which have been widely accepted. Defining

$$D_{BS} = C_{D_{\mathrm{wet}}} \frac{1}{2}\rho u^2 S_{\mathrm{wet}} \tag{A4}$$

$$C_{D_{\mathrm{wet}}} = C_{f_B}\left[1 + \frac{3}{2}\left(\frac{b}{a}\right)^{3/2} + 7\left(\frac{b}{a}\right)^3\right] \tag{A5}$$

‡Because of the exponential term, the integral converges well as t increases.

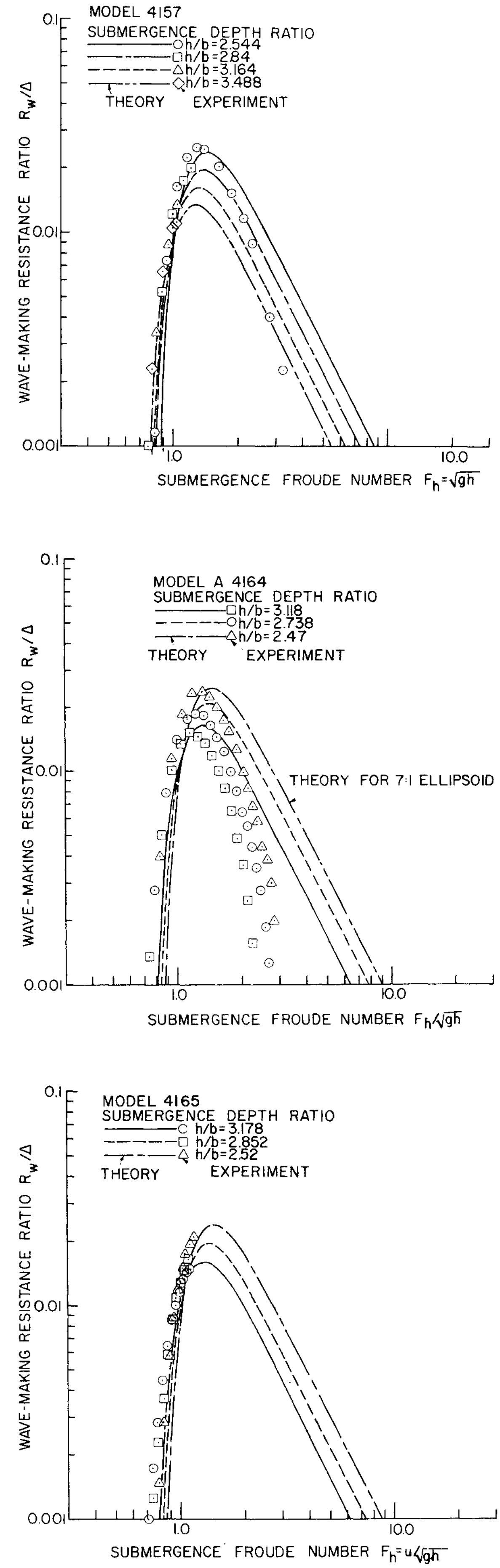

Fig. A1 Comparison between the theoretical wave drag of a 7:1 prolate ellipsoid and some experimental measurements

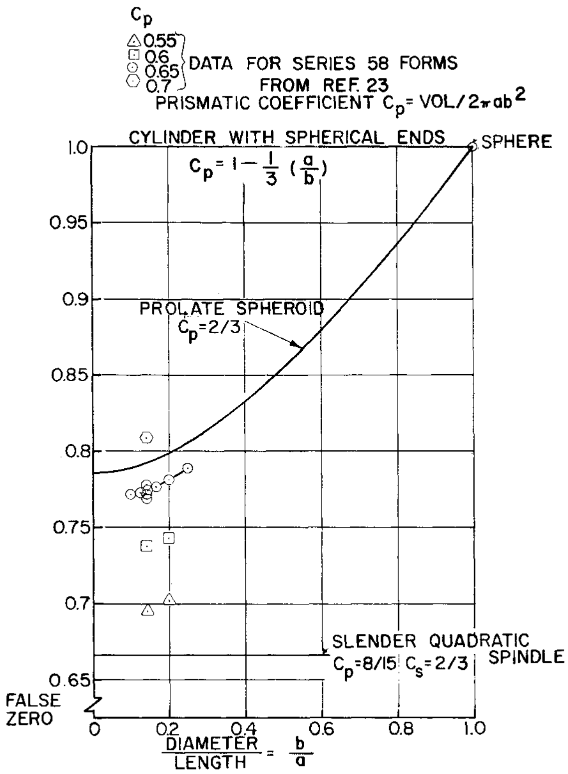

Fig. A2 Wetted area coefficient, as a function of fineness ratio and prismatic coefficient.

where C_{f_B} is the usual flat-plate skin-friction coefficient, which may be conveniently expressed as

$$C_{f_B} = \frac{0.427}{(\log_{10} R_{2a} - 0.407)^{2.64}} \tag{A6}$$

$$R_{2a} = 2au/\nu \text{ (Reynolds number based on length)} \tag{A7}$$

$$\nu \simeq 1.25 \times 10^{-5} \text{ (ft}^2\text{/s) for water at } 15°\text{C}$$

There is no simple relationship between the wetted area S_{wet} and b/a, because other factors are involved, principally the prismatic coefficient,

$$C_P = \frac{\text{volume}}{2\pi ab^2} \tag{A8}$$

A typical variation of

$$C_S = \frac{S_{wet}}{4\pi ab} \tag{A9}$$

with b/a and C_P as given in Fig. A2. C_s is obviously less at the lower prismatics; but then, so is the displacement, so this tends to cancel out. For the purpose of preliminary calculations, therefore, we might as well use the spheriod relationship

$$S_{wet} = 2\pi a^2 \left(\frac{b}{a}\right)\left(\frac{b}{a} + \frac{\sin^{-1}\epsilon}{\epsilon}\right) \tag{A10}$$

where

$$\epsilon = \sqrt{1-(b/a)^2}$$

Since we are generally working to a given displacement, the hull volume V is known, and since

$$V = \frac{4}{3}\pi a^3\left(\frac{b}{a}\right)^2 \quad a = \left(\frac{a}{b}\right)^{2/3}\left(\frac{3V}{4\pi}\right)^{1/3} \tag{A11}$$

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